OCR

Thursday 12 June 2014 – Afternoon

AS GCE MATHEMATICS

4728/01 Mechanics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4728/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 A particle P is projected vertically downwards with initial speed $3.5\,\mathrm{m\,s^{-1}}$ from a point A which is $5\,\mathrm{m}$ above horizontal ground.

(i) Find the speed of P immediately before it strikes the ground. **[2]**

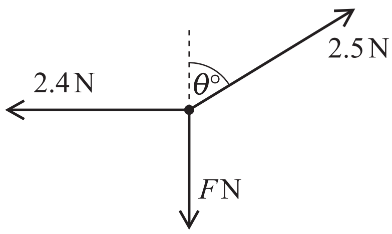
After striking the ground, P rebounds and moves vertically upwards and $0.87\,\mathrm{s}$ after leaving the ground P passes through A.

(ii) Calculate the speed of P immediately after it leaves the ground. **[3]**

It is given that the mass of P is $0.2\,\mathrm{kg}$.

(iii) Calculate the change in the momentum of P as a result of its collision with the ground. **[2]**

2



A particle rests on a smooth horizontal surface. Three horizontal forces of magnitudes $2.5\,\mathrm{N}$, $F\,\mathrm{N}$ and $2.4\,\mathrm{N}$ act on the particle on bearings $\theta°$, $180°$ and $270°$ respectively (see diagram). The particle is in equilibrium.

(i) Find θ and F. **[4]**

The $2.4\,\mathrm{N}$ force suddenly ceases to act on the particle, which has mass $0.2\,\mathrm{kg}$.

(ii) Find the magnitude and direction of the acceleration of the particle. **[3]**

3 A particle P travels in a straight line. The velocity of P at time t seconds after it passes through a fixed point A is given by $(0.6t^2 + 3)\,\mathrm{m\,s^{-1}}$. Find

(i) the velocity of P when it passes through A, **[1]**

(ii) the displacement of P from A when $t = 1.5$, **[4]**

(iii) the velocity of P when it has acceleration $6\,\mathrm{m\,s^{-2}}$. **[3]**

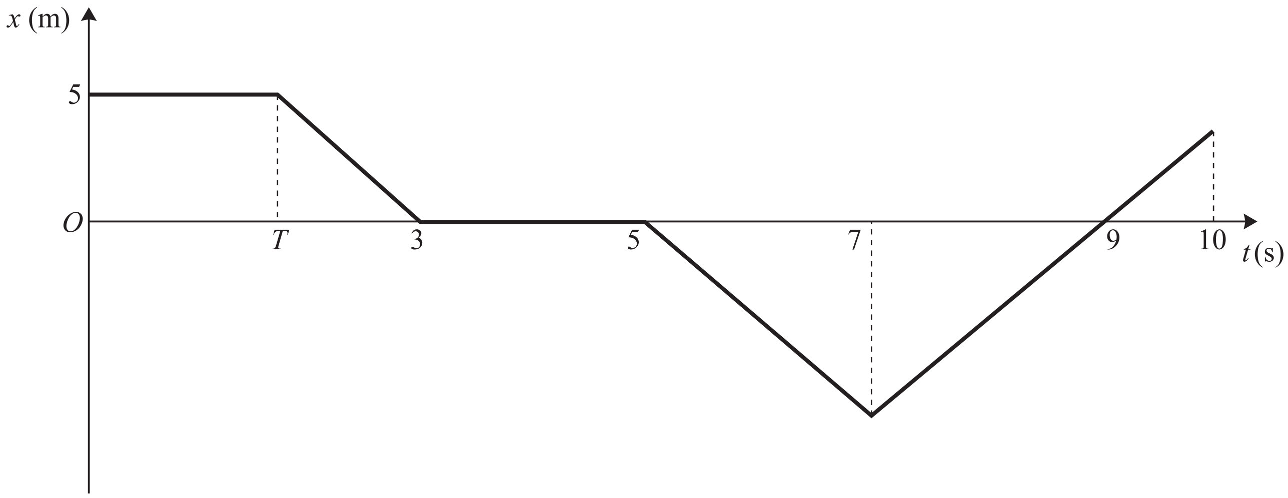
4

Particles P and Q are moving towards each other with constant speeds $4\,\text{m}\,\text{s}^{-1}$ and $2\,\text{m}\,\text{s}^{-1}$ along the same straight line on a smooth horizontal surface (see diagram). P has mass 0.2 kg and Q has mass 0.3 kg. The two particles collide.

(i) Show that Q must change its direction of motion in the collision. **[3]**

(ii) Given that P and Q move with equal speed after the collision, calculate both possible values for their speed after they collide. **[5]**

5



A particle P can move in a straight line on a horizontal surface. At time t seconds the displacement of P from a fixed point A on the line is x m. The diagram shows the (t, x) graph for P. In the interval $0 \leqslant t \leqslant 10$, either the speed of P is $4\,\text{m}\,\text{s}^{-1}$, or P is at rest.

(i) Show by calculation that $T = 1.75$. **[2]**

(ii) State the velocity of P when

(a) $t = 2$, **[1]**

(b) $t = 8$, **[1]**

(c) $t = 9$. **[1]**

(iii) Calculate the distance travelled by P in the interval $0 \leqslant t \leqslant 10$. **[3]**

For $t > 10$, the displacement of P from A is given by $x = 20t - t^2 - 96$.

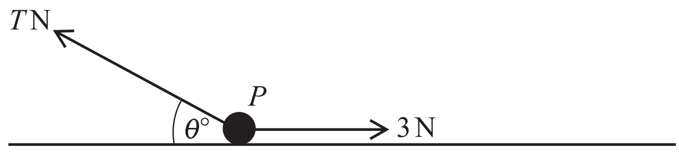
(iv) Calculate the value of t, where $t > 10$, for which the speed of P is $4\,\text{m}\,\text{s}^{-1}$. **[4]**

6 A particle P of weight 8 N rests on a horizontal surface. A horizontal force of magnitude 3 N acts on P, and P is in limiting equilibrium.

(i) Calculate the coefficient of friction between P and the surface. **[2]**

(ii) Find the magnitude and direction of the contact force exerted by the surface on P. **[4]**

(iii)

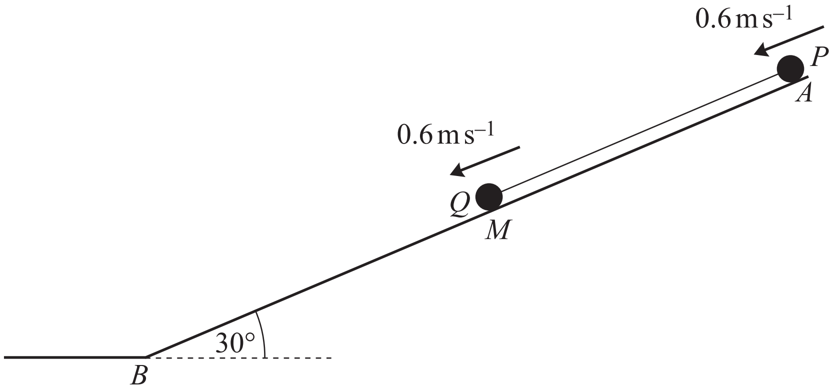


The initial 3 N force continues to act on P in its original direction. An additional force of magnitude T N, acting in the same vertical plane as the 3 N force, is now applied to P at an angle of $\theta°$ above the horizontal (see diagram). P is again in limiting equilibrium.

(a) Given that $\theta = 0$, find T. **[2]**

(b) Given instead that $\theta = 30$, calculate T. **[6]**

7



A and *B* are points at the upper and lower ends, respectively, of a line of greatest slope on a plane inclined at 30° to the horizontal. *M* is the mid-point of *AB*. Two particles *P* and *Q*, joined by a taut light inextensible string, are placed on the plane at *A* and *M* respectively. The particles are simultaneously projected with speed $0.6\,\text{m}\,\text{s}^{-1}$ down the line of greatest slope (see diagram). The particles move down the plane with acceleration $0.9\,\text{m}\,\text{s}^{-2}$. At the instant 2 s after projection, *P* is at *M* and *Q* is at *B*. The particle *Q* subsequently remains at rest at *B*.

(i) Find the distance *AB*. **[3]**

The plane is rough between *A* and *M*, but smooth between *M* and *B*.

(ii) Calculate the speed of *P* when it reaches *B*. **[4]**

P has mass 0.4 kg and *Q* has mass 0.3 kg.

(iii) By considering the motion of *Q*, calculate the tension in the string while both particles are moving down the plane. **[3]**

(iv) Calculate the coefficient of friction between *P* and the plane between *A* and *M*. **[6]**

END OF QUESTION PAPER

Question			Answer	Marks	Guidance	
1	(i)		$v^2 = 3.5^2 + 2g \times 5$ $v = 10.5$ ms^{-1}	M1 A1 [2]	Uses $v^2 = 3.5^2$ +/– $2g5$	Accept -3.5^2 for $(-3.5)^2$ etc
	(ii)		$5 = 0.87u - g \times 0.87^2/2$ $u = 10.0$ m s^{-1}	M1 A1 A1 [3]	+/-5 = $0.87u$ +/– $g\ 0.87^2/2$	May come from $s = vt - gt^2/2$
	(iii)		Change = $0.2 \times 10.5 + 0.2 \times 10$ Change = 4.1(0) kg m s^{-1}	M1 A1 [2]	Or +/- 0.2(Ans(i) +/– Ans(ii)) It is OK get -4.1 from correct work	
2	(i)		$2.5\sin\theta = 2.4$ $\theta = 73.7$ $2.5\cos\theta = F$ $F = 0.7$ *OR* $2.4^2+F^2=2.5^2$ *or* $F^2=2.5^2-2.4^2$ $F = 0.7$	M1 A1 M1 A1 [4] M1 A1	2.5CorSθ = 2.4 Accept 74 F = 2.5 SorCθ, opposite to that above Exact, but allow 0.702 (3 sf) $\theta = 73.7$	$2.5\cos\theta = 2.4$ M1 hence $\theta = 16.3$ A0 $2.5\sin\theta = F$ M1 hence F =0.7(00) A1 SC F can then be used to find θ
	(ii)		$2.4 = 0.2a$ $a = 12$ ms^{-2} Bearing (0)90° *OR* "To right"," opposite old 2.4 N force" etc	M1 A1 B1 [3]	N2L, Any horizontal force other than F, 0.7, 2.5 (Do not treat removing/using 2.5 as a MR) 12.0 from 2.5sin73.7 /0.2 Angle value other than exactly 90° or 0° B0 Allow B1 for force dirn, if accn not found	Including g, automatically M0 Horizontal is B0 (ambiguous)

Question			Answer	Marks	Guidance	
3	(i)		3 ms^{-1}	B1 [1]		MR $(0.6t^3 + 3)$, award B1 here
	(ii)		$x = \int 0.6t^2 + 3 \; dt$ $x = 0.6t^3/3 + 3t \; (+ c)$ Substitutes 1.5 in expression for x $x(1.5) = 5.175$ m	M1* A1 D*M1 A1 [4]	Integrates v Accept with/without $+ c$ Needs integration and 2 terms in t Only without +c. Accept 5.17, 5.18	MR $(0.6t^3 + 3)$ $0.6t^4/4 + 3t$ is A0 MR 5.26 only gets A1ft
	(iii)		$a = d(0.6t^2 + 3)/dt$ $6 = 2 \times 0.6t$ $v(5) = 18$ ms^{-1}	M1* D*M1 A1 [3]	Differentiates v Plus attempt to solve $a(t) = 6$	MR $(0.6t^3 + 3)$ gives t=1.82(57..) v(1.8257..) = 6.65 (3 sf)
4	(i)		Calculation for both "before" Momentum (magnitudes) Compares both terms without arithmetic error Shows direction of after total momentum conflicts with the before velocity/momentum of Q	M1 A1* D*A1 [3]	Must not include g Vector nature of momentum by word or sign (+/-)	 Explicit reference to after momentum or conservation of momentum essential.
	(ii)		TMB = +/-$(0.2 \times 4 + 0.3 \times (-2))$ $0.8 - 0.6 = 0.2v + 0.3v$ $v = 0.4$ m s^{-1} $0.8 - 0.6 = -0.2v + 0.3v$ $v = 2$ m s^{-1}	B1 M1 A1 M1 A1 [5]	Accept inclusion of g Allow if g included in all terms Not awarded if g included Allow if g included in all terms Not awarded if g included	 LHS must be difference for both M1 marks SC $0.8 - 0.6 = 0.2v - 0.3v$ M1 Speed = 2 **and** the direction of motion of Q is reversed A1

Question			Answer	Marks	Guidance	
5	(i)		$5/(T-3)=-4$ *OR* $5/(3-T)=4$ $T=1.75$	M1 A1 [2]	Accept verification, $4\times(3-1.75)$ M1 = 5 A1 *OR* 5/(3-1.75) M1 = 4 A1	
	(ii)	(a)	$-4\ \text{ms}^{-1}$	B1 [1]		
		(b)	$4\ \text{ms}^{-1}$	B1 [1]		
		(c)	$4\ \text{ms}^{-1}$	B1 [1]		
	(iii)		$2\times(-)4,\ 2\times4,\ (1\times)4$ $d=(-)5+(-)8+8+4$ $d=25$ m	M1* D*M1 A1 [3]	Calculates any one unknown distance Adds 5 and "3 other" distances or -5 and "3 other" displacements Correctly comes from 4x(1.25+4+1) 3/3	Allow if only one calc. correct Note t=5 to t=9, t=5 to t=10 etc, may be one term
	(iv)		$v=\text{d}(20t-t^2-96)/\text{d}t$ $v=20-2t$ $20-2t=-4$ $t=12$ (ignore any solutions less than 10)	M1* A1 D*M1 A1 [4]	Differentiates x, accept $20-t$ as "differentiation" $20-2t+c=-4$ is DM0 Only from $20-2t=-4$. This answer can arise fortuitously from solving $20t-t^2-96=0$.	SC Verifying that t=12 gives v= -4 can gain final M1A1 (A special case of trial and refinement)

Question			Answer	Marks	Guidance	
6	(i)		$3 = 8\mu$ $\mu = 0.375$	M1 A1 [2]	Uses $F = \mu R$, Allow R is 8 or $8g$, Fr =3 only 3/8 (fraction), not 3÷8 (division)	
	(ii)		$C^2 = 3^2 + 8^2$ $C = 8.54$ N $\tan\theta = 3/8$ or $\tan\theta = 8/3$ $\theta = 20.6°$ with vertical or 69.4° with horizontal	M1 A1 M1 A1 [4]	Uses Pythagoras with 3 and 8 or $8g$ Accept 8.5 or √73 Uses tan with 3 and 8 or $8g$ Accept 21 or 69, direction clear by words or diagram.	 Or CorS with answer for C isw work after correct angle magnitude found
	(iii)	(a)	$T(\cos 0) - 3 = +/-3$ $T = 6$	M1 A1 [2]	$T(\cos 0) - 3 = 0$ is M0 Answer alone is sufficient for M1A1	$T\cos 0 - 3 = -3$ assumes Fr direction has not changed
	(iii)	(b)	$R = +/-(8 - T \times \text{SorC}30)$ $R = 8 - T\sin 30$ $Fr = +/- (T \times \text{CorS}30 - 3)$ $Fr = T\cos 30 - 3$ $0.375 = (T\cos 30 - 3) / (8 - T\sin 30)$ $T = 5.70$ *OR* Alternative for last 4 marks $Fr = 0.375(8 - T\sin 30)$ $Fr = +/- (T \times \text{CorS}30 - 3)$ $Fr = T\cos 30 - 3$ $0.375(8 - T\sin 30) = T\cos 30 - 3$ $T = 5.70$	M1 A1 M1 A1 M1 A1 [6] - M1 A1 M1 A1	Accept $8g$ with cmpt T oe Accept 3 with cmpt T, not T×CorS30+/–3= 0 oe Accept use of μ from (i). For forming an equation in T alone. Accept use of μ from (i). oe For forming an equation in T alone.	(This is required also in the SC case) SC Does not allow for change in direction of Friction $Fr = 3 - T\cos 30$ A1 $0.375 = (3 - T\cos 30) / (8 - T\sin 30)$ M1 $T = 0$ A0 SC (Alternative) $Fr = 0.375(8 - T\sin 30)$ $Fr = +/- (T \times \text{CorS}30 - 3)$ M1 $Fr = 3 - T\cos 30$ A1 $0.375(8 - T\sin 30) = (3 - T\cos 30)$ M1 $T = 0$ A0

Question			Answer	Marks	Guidance	
7	(i)		$s = 0.6 \times 2 + 0.9 \times 2^2/2$	M1	Uses $s = ut + at^2/2$, $u \neq 0$, $a \neq g$ or gCorS30	
			$s = 3$	A1		
			$AB = 6$ m	A1		
				[3]		
	(ii)		$V_M = 0.6 + 0.9 \times 2$ *OR*		2.4	Award if found in (i) and used in (ii)
			$V_M{}^2 = 0.6^2 + 2\text{x}0.9\times3$	B1	5.76	
			$a = g\sin30$	B1	4.9	
			$V_B{}^2 = 2.4^2 + 2(9.8\sin30) \times 3$	M1	Uses $v^2 = u^2 + 2as$, $u \neq 0$ or 0.6, $a \neq g$ or 0.9, $s \neq AB$(i)	If AB(i) = 3, allow its use for final M1A1
			$V_B = 5.93$ ms^{-1}	A1	Accept 5.9	
				[4]		
	(iii)			M1	N2L, $0.3 \times 0.9 = +/-(0.3\text{gCorS30} - T)$	$a = 0.9$ essential, $m = 0.3$ but if 0.4 used in (iii) AND 0.3 used in (iv), treat as a single mis-read
			$0.3 \times 0.9 = 0.3g\sin30 - T$	A1		
			$T = 1.2$ N	A1		
				[3]		
	(iv)			M1*	N2L, 3 forces inc $+/-(0.4g\text{CorS30} + T)$	$a = 0.9$ or value used in (iii), m=0.4 but if 0.4 used in (iii) AND 0.3 used in (iv), treat as a single mis-read
			$0.4 \times 0.9 = 0.4g\sin30 + 1.2 - Fr$	A1ft	ft cv(T) in (iii)	
			$Fr = 2.8$	A1	May be shown by mu calculation	
			$R = 0.4g\cos 30$	B1	May be implied, 3.39(48...)	
			$\mu = 2.8/3.39$	D*M1	$2.8 = 3.39(48)\ \mu$, both forces positive	Awarded only if M1 forN2L equation
			$\mu = 0.825$	A1	Accept 0.82, not 0.83 or 0.826	
				[6]		